2.4 Exercises

1–10 ■ Suppose the graph of f is given. Describe how the graph of each function can be obtained from the graph of f.

1. **(a)** $y = f(x) - 5$ **(b)** $y = f(x - 5)$

2. **(a)** $y = f(x + 7)$ **(b)** $y = f(x) + 7$

3. **(a)** $y = f(x + \frac{1}{2})$ **(b)** $y = f(x) + \frac{1}{2}$

4. **(a)** $y = -f(x)$ **(b)** $y = f(-x)$

5. **(a)** $y = -2f(x)$ **(b)** $y = -\frac{1}{2}f(x)$

6. **(a)** $y = -f(x) + 5$ **(b)** $y = 3f(x) - 5$

7. **(a)** $y = f(x - 4) + \frac{3}{4}$ **(b)** $y = f(x + 4) - \frac{3}{4}$

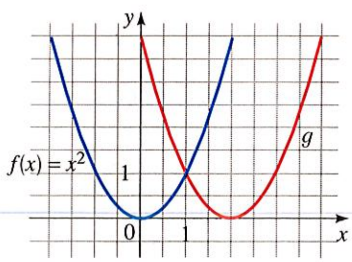
8. **(a)** $y = 2f(x + 2) - 2$ **(b)** $y = 2f(x - 2) + 2$

9. **(a)** $y = f(4x)$ **(b)** $y = f(\frac{1}{4}x)$

10. **(a)** $y = -f(2x)$ **(b)** $y = f(2x) - 1$

11–16 ■ The graphs of f and g are given. Find a formula for the function g.

11.



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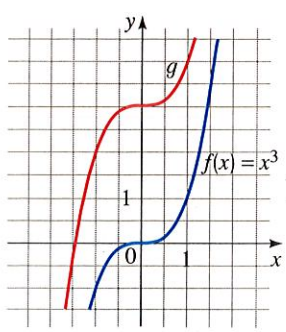

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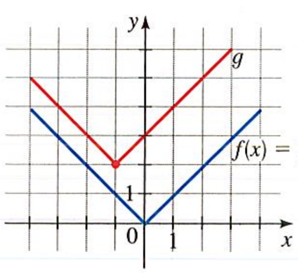

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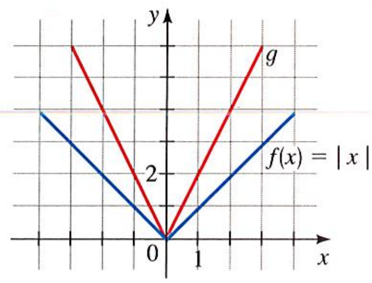

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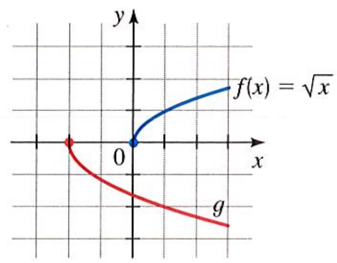

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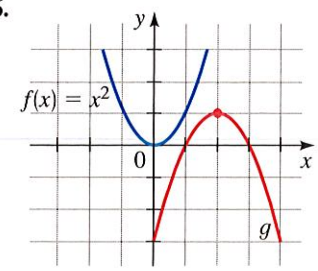
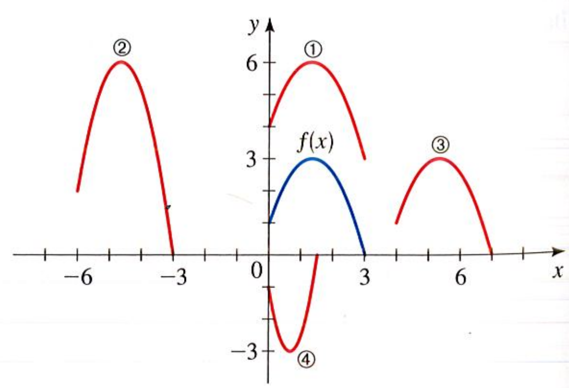


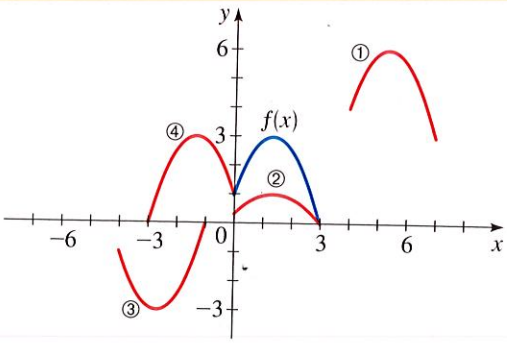
17–18 ■ The graph of $y = f(x)$ is given. Match each equation with its graph.

17. **(a)** $y = f(x - 4)$ **(b)** $y = f(x) + 3$

(c) $y = 2f(x + 6)$ **(d)** $y = -f(2x)$

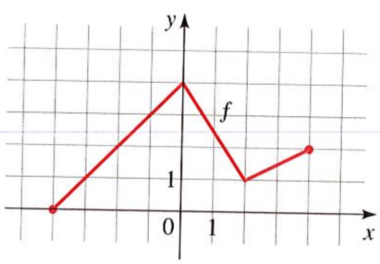


18. **(a)** $y = \frac{1}{3}f(x)$ **(b)** $y = -f(x + 4)$
(c) $y = f(x - 4) + 3$ **(d)** $y = f(-x)$



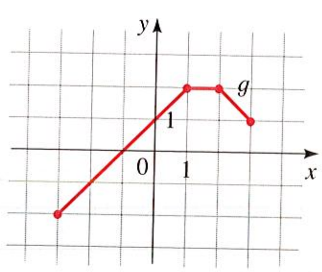
19. The graph of f is given. Sketch the graphs of the following functions.

(a) $y = f(x - 2)$ **(b)** $y = f(x) - 2$
(c) $y = 2f(x)$ **(d)** $y = -f(x) + 3$
(e) $y = f(-x)$ **(f)** $y = \frac{1}{2}f(x - 1)$



20. The graph of g is given. Sketch the graphs of the following functions.

(a) $y = g(x + 1)$ **(b)** $y = -g(x + 1)$
(c) $y = g(x - 2)$ **(d)** $y = g(x) - 2$
(e) $y = -g(x) + 2$ **(f)** $y = 2g(x)$



21. **(a)** Sketch the graph of $f(x) = \dfrac{1}{x}$ by plotting points.

(b) Use the graph of f to sketch the graphs of the following functions.

(i) $y = -\dfrac{1}{x}$ (ii) $y = \dfrac{1}{x - 1}$

(iii) $y = \dfrac{2}{x + 2}$ (iv) $y = 1 + \dfrac{1}{x - 3}$

22. **(a)** Sketch the graph of $g(x) = \sqrt[3]{x}$ by plotting points.

(b) Use the graph of g to sketch the graphs of the following functions.

(i) $y = \sqrt[3]{x - 2}$ (ii) $y = \sqrt[3]{x + 2} + 2$
(iii) $y = 1 - \sqrt[3]{x}$ (iv) $y = 2\sqrt[3]{x}$

23–26 ■ Explain how the graph of g is obtained from the graph of f.

23. **(a)** $f(x) = x^2$, $g(x) = (x + 2)^2$
(b) $f(x) = x^2$, $g(x) = x^2 + 2$

24. **(a)** $f(x) = x^3$, $g(x) = (x - 4)^3$
(b) $f(x) = x^3$, $g(x) = x^3 - 4$

25. **(a)** $f(x) = \sqrt{x}$, $g(x) = 2\sqrt{x}$
(b) $f(x) = \sqrt{x}$, $g(x) = \frac{1}{2}\sqrt{x - 2}$

26. **(a)** $f(x) = |x|$, $g(x) = 3|x| + 1$
(b) $f(x) = |x|$, $g(x) = -|x + 1|$

27–32 ■ A function f is given, and the indicated transformations are applied to its graph (in the given order). Write the equation for the final transformed graph.

27. $f(x) = x^2$; shift upward 3 units and shift 2 units to the right

28. $f(x) = x^3$; shift downward 1 unit and shift 4 units to the left

29. $f(x) = \sqrt{x}$; shift 3 units to the left, stretch vertically by a factor of 5, and reflect in the x-axis

30. $f(x) = \sqrt[3]{x}$; reflect in the y-axis, shrink vertically by a factor of $\frac{1}{2}$, and shift upward $\frac{3}{5}$ unit

31. $f(x) = |x|$; shift to the right $\frac{1}{2}$ unit, shrink vertically by a factor of 0.1, and shift downward 2 units

32. $f(x) = |x|$; shift to the left 1 unit, stretch vertically by a factor of 3, and shift upward 10 units

33–48 ■ Sketch the graph of the function, not by plotting points, but by starting with the graph of a standard function and applying transformations.

33. $f(x) = (x - 2)^2$ **34.** $f(x) = (x + 7)^2$
35. $f(x) = -(x + 1)^2$ **36.** $f(x) = 1 - x^2$
37. $f(x) = x^3 + 2$ **38.** $f(x) = -x^3$
39. $y = 1 + \sqrt{x}$ **40.** $y = 2 - \sqrt{x + 1}$
41. $y = \frac{1}{2}\sqrt{x + 4} - 3$ **42.** $y = 3 - 2(x - 1)^2$
43. $y = 5 + (x + 3)^2$ **44.** $y = \frac{1}{3}x^3 - 1$
45. $y = |x| - 1$ **46.** $y = |x - 1|$
47. $y = |x + 2| + 2$ **48.** $y = 2 - |x|$

49–52 ■ Graph the functions on the same screen using the given viewing rectangle. How is each graph related to the graph in part (a)?

49. Viewing rectangle $[-8, 8]$ by $[-2, 8]$

(a) $y = \sqrt[4]{x}$ **(b)** $y = \sqrt[4]{x + 5}$

(c) $y = 2\sqrt[4]{x + 5}$ **(d)** $y = 4 + 2\sqrt[4]{x + 5}$

50. Viewing rectangle $[-8, 8]$ by $[-6, 6]$

(a) $y = |x|$ **(b)** $y = -|x|$

(c) $y = -3|x|$ **(d)** $y = -3|x - 5|$

51. Viewing rectangle $[-4, 6]$ by $[-4, 4]$

(a) $y = x^6$ **(b)** $y = \frac{1}{3}x^6$

(c) $y = -\frac{1}{3}x^6$ **(d)** $y = -\frac{1}{3}(x - 4)^6$

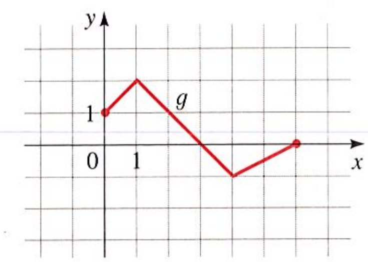
52. Viewing rectangle $[-6, 6]$ by $[-4, 4]$

(a) $y = \dfrac{1}{\sqrt{x}}$ **(b)** $y = \dfrac{1}{\sqrt{x + 3}}$

(c) $y = \dfrac{1}{2\sqrt{x + 3}}$ **(d)** $y = \dfrac{1}{2\sqrt{x + 3}} - 3$

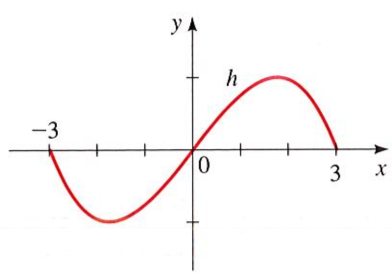
53. The graph of g is given. Use it to graph each of the following functions.

(a) $y = g(2x)$ **(b)** $y = g\left(\frac{1}{2}x\right)$



54. The graph of h is given. Use it to graph each of the following functions.

(a) $y = h(3x)$ **(b)** $y = h\left(\frac{1}{3}x\right)$

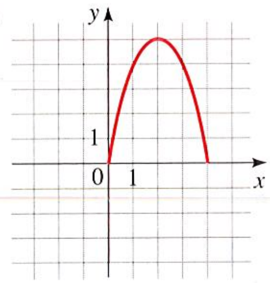


55–56 ■ The graph of a function defined for $x \geq 0$ is given. Complete the graph for $x < 0$ to make

(a) an even function

(b) an odd function

55.



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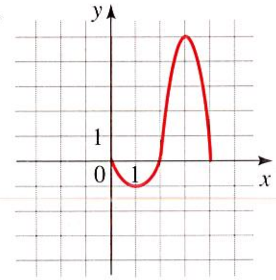


57–58 ■ Use the graph of $f(x) = [\![x]\!]$ described on pages 162–163 to graph the indicated function.

57. $y = [\![2x]\!]$ **58.** $y = [\![\frac{1}{4}x]\!]$

59. If $f(x) = \sqrt{2x - x^2}$, graph the following functions in the viewing rectangle $[-5, 5]$ by $[-4, 4]$. How is each graph related to the graph in part (a)?

(a) $y = f(x)$ **(b)** $y = f(2x)$ **(c)** $y = f\left(\frac{1}{2}x\right)$

60. If $f(x) = \sqrt{2x - x^2}$, graph the following functions in the viewing rectangle $[-5, 5]$ by $[-4, 4]$. How is each graph related to the graph in part (a)?

(a) $y = f(x)$ **(b)** $y = f(-x)$ **(c)** $y = -f(-x)$

(d) $y = f(-2x)$ **(e)** $y = f\left(-\frac{1}{2}x\right)$

61–68 ■ Determine whether the function f is even, odd, or neither. If f is even or odd, use symmetry to sketch its graph.

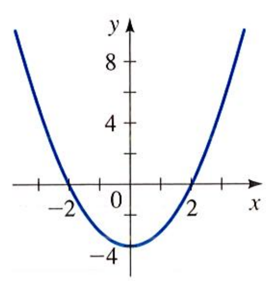
61. $f(x) = x^{-2}$ **62.** $f(x) = x^{-3}$

63. $f(x) = x^2 + x$ **64.** $f(x) = x^4 - 4x^2$

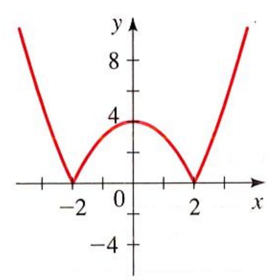
65. $f(x) = x^3 - x$ **66.** $f(x) = 3x^3 + 2x^2 + 1$

67. $f(x) = 1 - \sqrt[3]{x}$ **68.** $f(x) = x + \dfrac{1}{x}$

69. The graphs of $f(x) = x^2 - 4$ and $g(x) = |x^2 - 4|$ are shown. Explain how the graph of g is obtained from the graph of f.

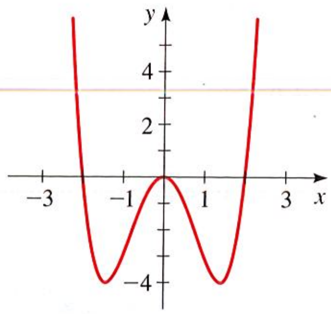


$f(x) = x^2 - 4$



$g(x) = |x^2 - 4|$

70. The graph of $f(x) = x^4 - 4x^2$ is shown. Use this graph to sketch the graph of $g(x) = |x^4 - 4x^2|$.



71–72 ■ Sketch the graph of each function.

71. (a) $f(x) = 4x - x^2$ **(b)** $g(x) = |4x - x^2|$

72. (a) $f(x) = x^3$ **(b)** $g(x) = |x^3|$

Applications

73. Sales Growth The annual sales of a certain company can be modeled by the function $f(t) = 4 + 0.01t^2$, where t represents years since 1990 and $f(t)$ is measured in millions of dollars.

(a) What shifting and shrinking operations must be performed on the function $y = t^2$ to obtain the function $y = f(t)$?

(b) Suppose you want t to represent years since 2000 instead of 1990. What transformation would you have to apply to the function $y = f(t)$ to accomplish this? Write the new function $y = g(t)$ that results from this transformation.

74. Changing Temperature Scales The temperature on a certain afternoon is modeled by the function

$$C(t) = \tfrac{1}{2}t^2 + 2$$

where t represents hours after 12 noon $(0 \le t \le 6)$, and C is measured in °C.

(a) What shifting and shrinking operations must be performed on the function $y = t^2$ to obtain the function $y = C(t)$?

(b) Suppose you want to measure the temperature in °F instead. What transformation would you have to apply to the function $y = C(t)$ to accomplish this? (Use the fact that the relationship between Celsius and Fahrenheit degrees is given by $F = \frac{9}{5}C + 32$.) Write the new function $y = F(t)$ that results from this transformation.

Discovery • Discussion

75. Sums of Even and Odd Functions If f and g are both even functions, is $f + g$ necessarily even? If both are odd, is their sum necessarily odd? What can you say about the sum if one is odd and one is even? In each case, prove your answer.

76. Products of Even and Odd Functions Answer the same questions as in Exercise 75, except this time consider the *product* of f and g instead of the sum.

77. Even and Odd Power Functions What must be true about the integer n if the function

$$f(x) = x^n$$

is an even function? If it is an odd function? Why do you think the names "even" and "odd" were chosen for these function properties?

2.5 Quadratic Functions; Maxima and Minima

A maximum or minimum value of a function is the largest or smallest value of the function on an interval. For a function that represents the profit in a business, we would be interested in the maximum value; for a function that represents the amount of material to be used in a manufacturing process, we would be interested in the minimum value. In this section we learn how to find the maximum and minimum values of quadratic and other functions.